Tidal disruption and astrophysical phenomena

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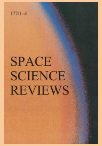
Outline

- The importance of tidal forces
- Theory and observables
- Basic formalism
- Theory meets observations : Kerr black holes
- Summary

Based on (+ references)

- ArXiv : 2401.17031 [astro-ph.HE]
- ArXiv : 2310.03539 [astro-ph.HE]
- JCAP 11 (2023) 062
- ApJ 929 (2022) 117
- ApJ 924 (2022) 20
- ApJ 910 (2021) 23
- ApJ 884 (2019) 95
- Space Science Review (2021) Jonker et al. Editors



Pritam Banerjee
Shaswata Chowdhury
Debojyoti Garain
Aryabrat Mahapatra
Adarsh Pandey

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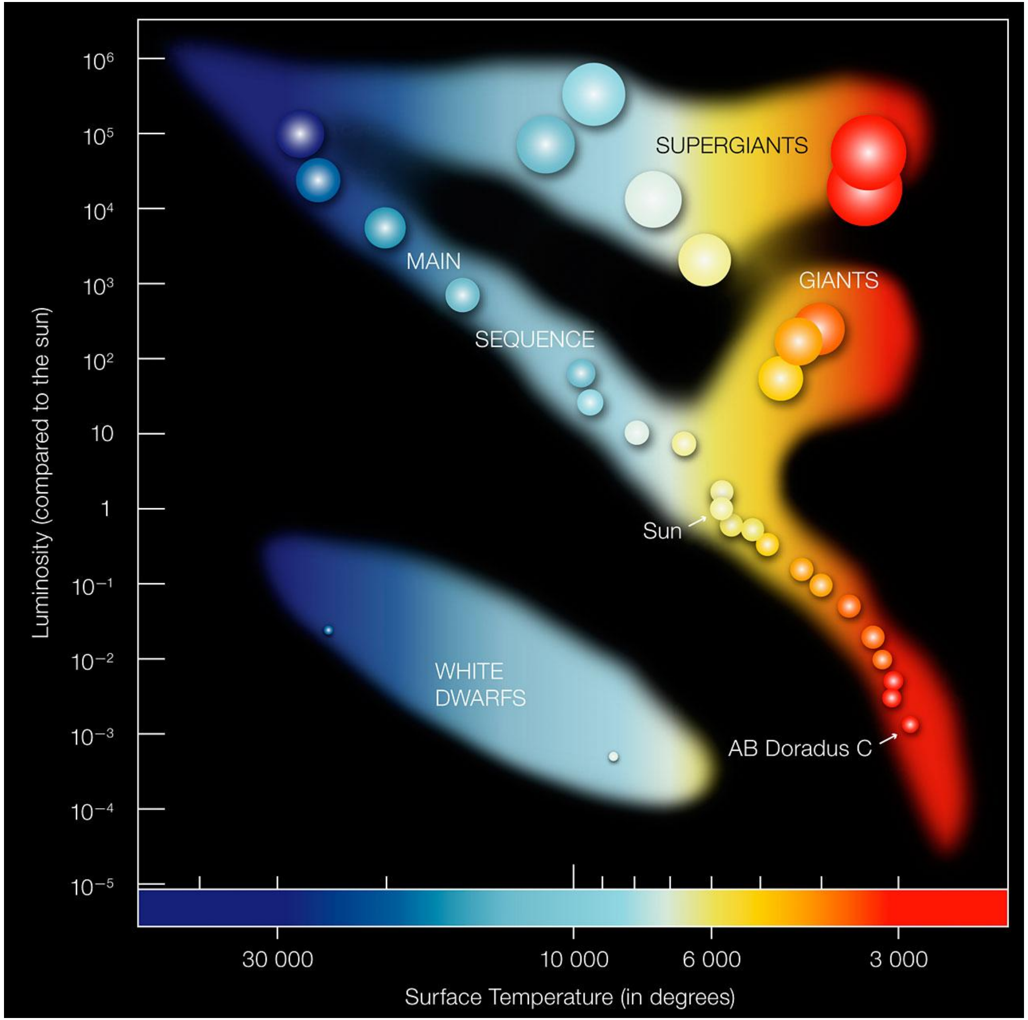


Hertzsprung Russell Diagram (Image Credit : ESO)

Due to internal mechanisms, stars of the main sequence will climb up the HR diagram, reach a giant phase and ultimately a red giant sheds much of its mass and becomes a white dwarf

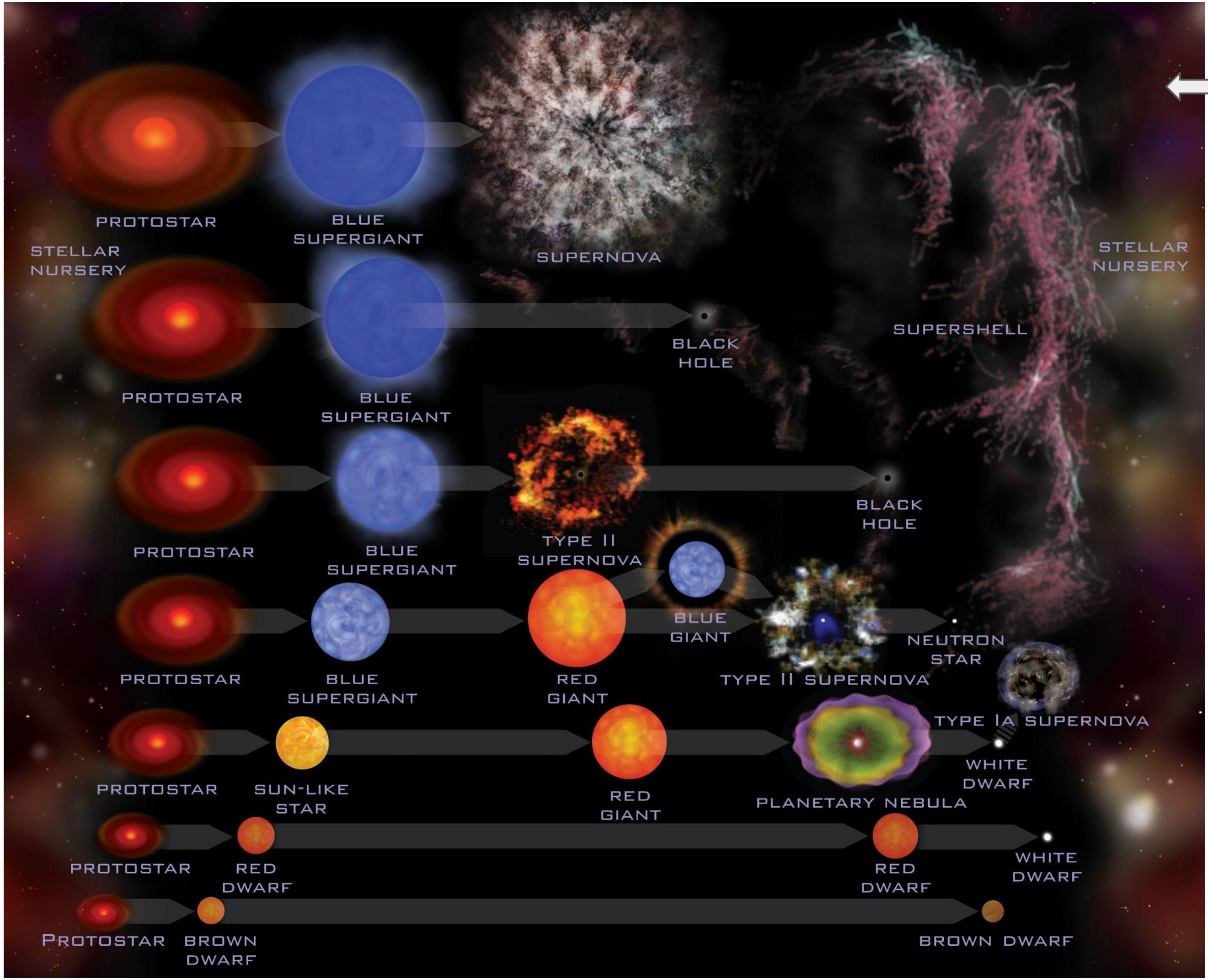


Image Credit : JPL

These are natural outcomes

A more dramatic situation might arise if a star meets a likely predator !

Sufficiently ferocious predators like black holes can eat up a star or rip it apart causing its unnatural death

This can happen due to non-local gravity : tides. Dramatic astrophysical phenomena.

The importance of tidal effects

Tidal effects are one of the most dramatic features of non-local gravity → can tear apart a massive object when non local forces exceed self gravity → can produce observable luminous flares → one of the main physical processes in accretion disc formation.

The comet Shoemaker Levy 9 broke into 21 pieces in July 1992 due to the tidal force of Jupiter and collided with it in July 1994 causing huge impacts on the surface of Jupiter.

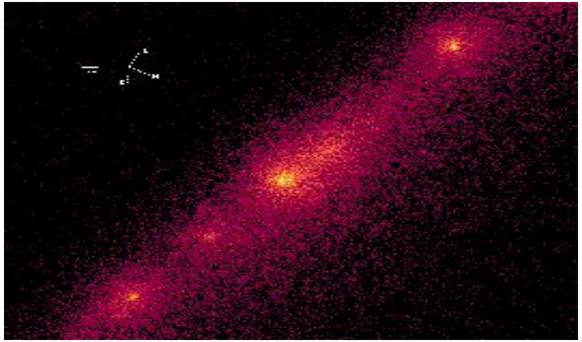
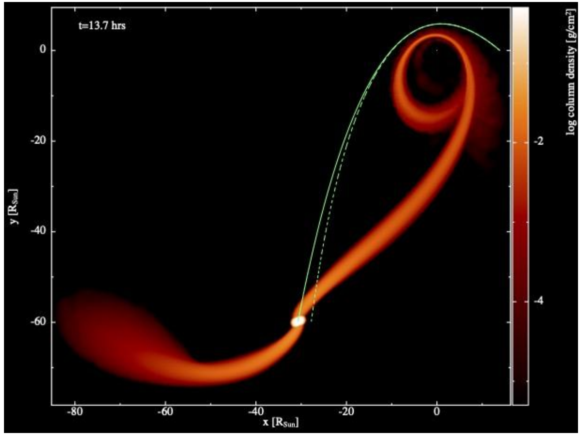


Image Credit : Wikimedia

When a star gets ripped apart due to tidal forces, one can get a fascinating view of its “interior” which is crucial to understanding stellar composition and stellar evolution : fallback rates are observable.

Provide astrophysical tests of gravity beyond GR.

Analytical computations not possible : this is a sample of how a tidal disruption event of a star looks like :



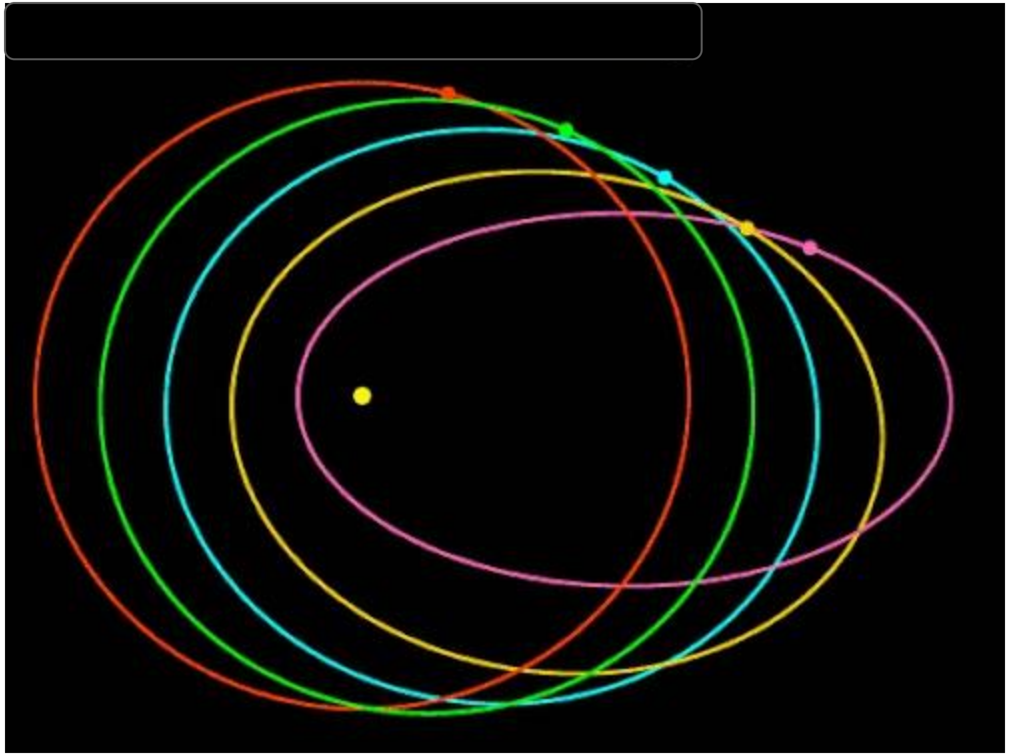
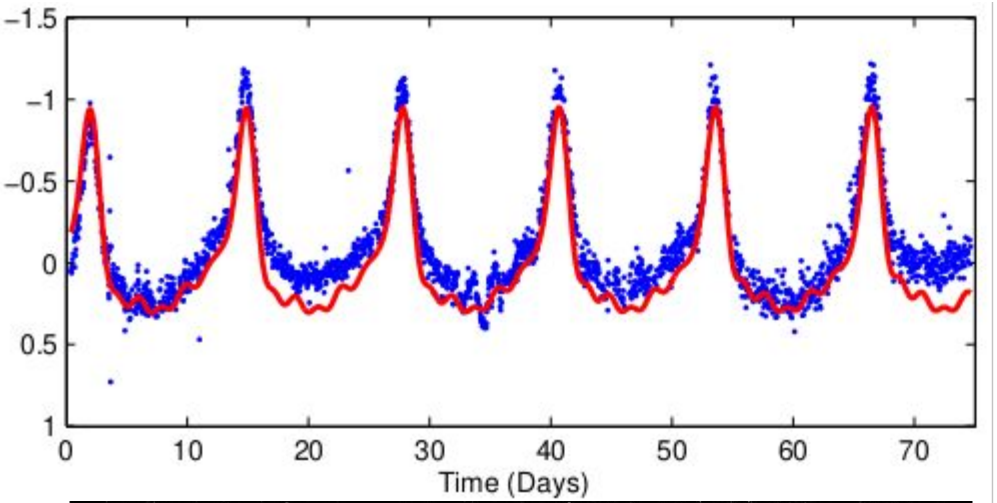


Image credit : Wikipedia

Tides not only cause disruptions, it can cause love too ! Called Heartbeat stars or stars in love

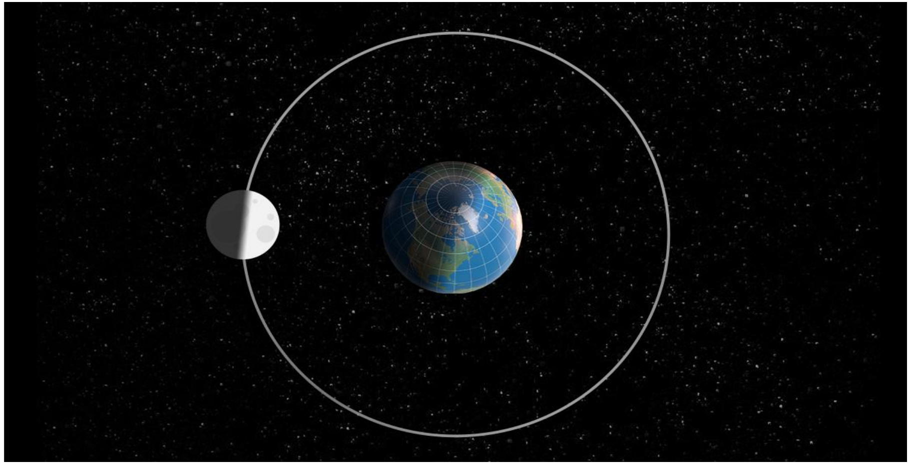
Image Credit : DST, GOI



Tidal torques play a major role in planetary systems

Can give rise to phenomena in planet-satellite systems like tidal locking

Image Credit : NASA



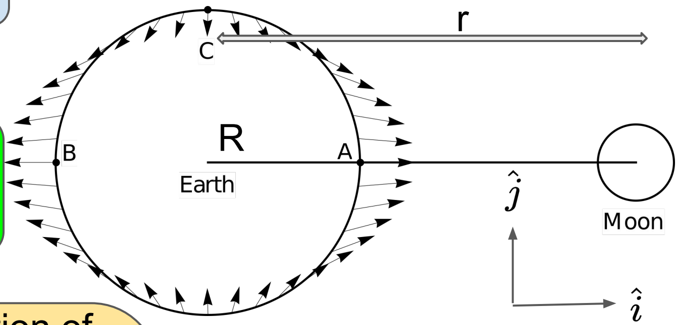
Familiar examples

- Earth tides. Can quantify tidal effects in simple scenarios :

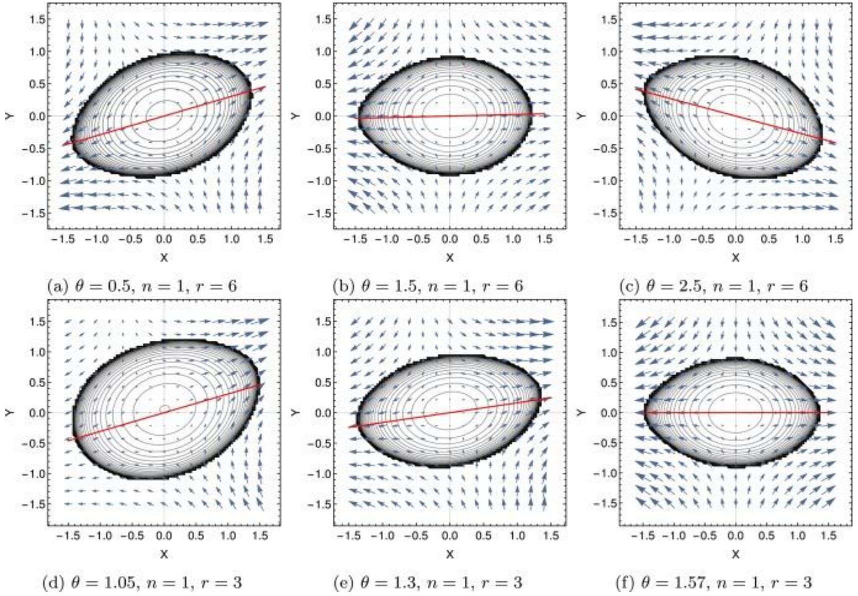
$$a_T(A) = \left[\frac{GM_m}{(r-R)^2} - \frac{GM_m}{r^2}\right]\hat{i} \simeq \frac{2GM_mR}{r^3}\hat{i}$$

$$a_T(C) \simeq -\frac{GM_mR}{r^3}\hat{j}$$

$$a_T(B) = \left[\frac{GM_m}{(r+R)^2} - \frac{GM_m}{r^2}\right]\hat{i} \simeq -\frac{2GM_mR}{r^3}\hat{i}$$



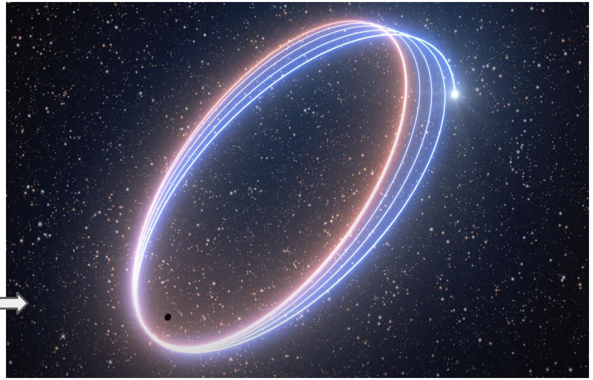
Stellar Tides



(a) $\theta = 0.5,\ n = 1,\ r = 6$ (b) $\theta = 1.5,\ n = 1,\ r = 6$ (c) $\theta = 2.5,\ n = 1,\ r = 6$

(d) $\theta = 1.05,\ n = 1,\ r = 3$ (e) $\theta = 1.3,\ n = 1,\ r = 3$ (f) $\theta = 1.57,\ n = 1,\ r = 3$

The deformation of the star gives rise to very interesting effects like tidal torques that maximise at periapsis and couples with tidal oscillations. Image Courtesy : Science



First take : Fermi Normal frame

- We need a frame of reference that is locally flat all along a timelike geodesic : Fermi normal frame.
- So we take a star in this local tetrad basis which moves with the star along its timelike geodesic $\mathcal{G}$.

- Local Flatness : An observer on $\mathcal{G}$ sees the flat Minkowski metric :

$$g_{\mu'\nu'}\hat{e}^{\mu'}_{\mu}\hat{e}^{\nu'}_{\nu} = \eta_{\mu\nu} \quad ; \qquad \Gamma^{\mu}_{\alpha\beta}\Big|_{\mathcal{G}} = 0$$

- Riemann Tensor remains non-zero.

- Valid throughout the geodesic : The basis vectors $\hat{e}^{\mu'}_{\nu}$ are to be parallel transported along the geodesic.

$$(\nabla_{\alpha'}\hat{e}^{\mu'}_{\nu})u^{\alpha'} = 0$$

Strategy :

For a generic stationary space-time, construct FNC.

⇩

Consider equatorial parabolic (or elliptical) orbits by suitably choosing energy and angular momentum.

⇩

Solve the hydrodynamic equation of the fluid star numerically in the presence of tidal potential.

⇩

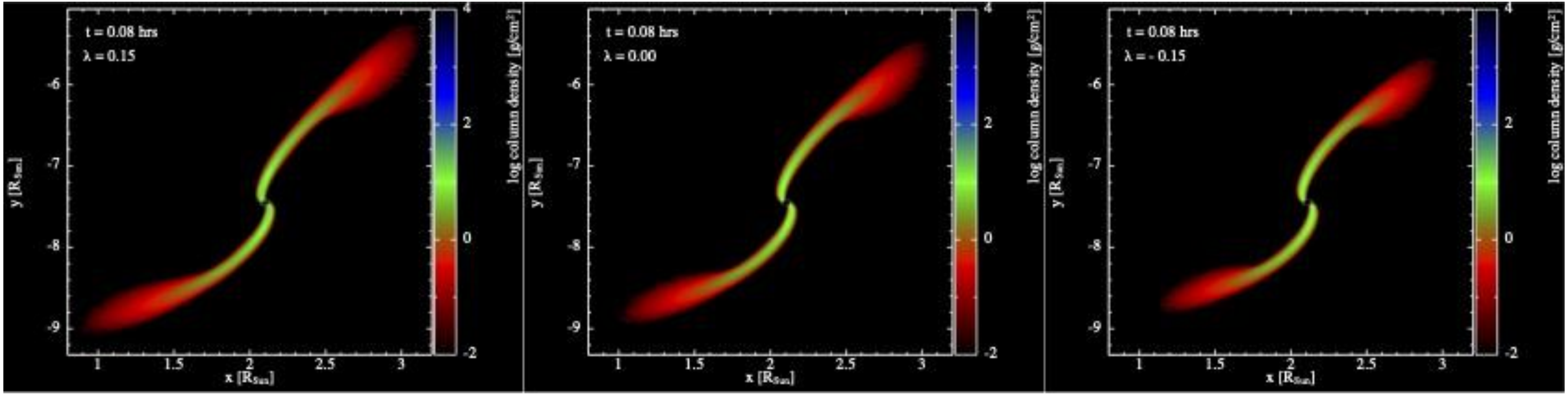
Obtain critical density below which star is disrupted. Gives other critical parameter values.

All this is great, but lacks dynamics : we always assumed hydrodynamic equilibrium.

Tidal disruption events can only be properly understood if we can follow an object in real (Newtonian) time as it approaches a central mass.

Stars are often partially disrupted : interesting physics that cannot be captured by equilibrium model.

Need high performance computing based on GR : SPH based parallel code



There are several parameters that need to be specified :

- Black hole mass
- Black hole spin
- Stellar mass
- Stellar radius
- Stellar spin
- Equation of state
- Stellar orbit (eccentricity) and periastron

- Black hole mass and spin can be estimated with reasonable accuracy. Usually categorised as stellar mass $\leq 10^2 M_\odot$ Intermediate mass $10^2 - 10^5 M_\odot$ supermassive $> 10^5 M_\odot$

- No spherical symmetry
- Star may be in off-equatorial orbit
- Spin of black hole may not align with spin of star
- Stellar equation of state has degeneracy
- Stellar orbit may not be possible to track
- Large stellar spin typically associated with large magnetic fields
- Disruption may be full or partial in elliptic or parabolic orbits

$$R_t = R_\star \left(\frac{M_{BH}}{M_\star}\right)^{1/3} ; \quad \beta = \frac{R_t}{R_p}$$

White dwarfs are swallowed by SMBHs as $R_t < R_S$
White dwarf EOS is reasonably accurate.

$$t_{\min} = \left(\frac{M_{BH}}{10^6 M_\odot}\right)^{1/2} \left(\frac{M_\star}{M_\odot}\right)^{-1} \left(\frac{R_\star}{R_\odot}\right)^{3/2} \times 40 \ \mathbf{day} \Longrightarrow \dot{M} = \frac{M_\star}{3t_{\min}} \left(\frac{t}{t_{\min}}\right)^{-5/3}$$

In partial tidal disruptions by IMBHs, this formula is modified.

Let us see how things work for Schwarzschild : Point mass moving in timelike geodesic

$$ds^2 = -\left(1 - \frac{2r_g}{r}\right)c^2dt^2 + \left(1 - \frac{2r_g}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin\theta^2 d\varphi^2)$$

$$\xi_t^\mu = \{1,0,0,0\}, \quad \xi_\phi^\mu = \{0,0,0,1\}$$

$$\epsilon = c^2\left(1 - \frac{2r_g}{r}\right)\frac{dt}{d\tau}, \quad l = r^2\frac{d\varphi}{d\tau}$$

Normalise $u^\mu u_\mu = -c^2$ $\Rightarrow$

$$\left(\frac{dr}{d\tau}\right)^2 = \left(\frac{\epsilon^2 - c^4}{c^2}\right) - \left[\frac{l^2}{r^2}\left(1 - \frac{2r_g}{r}\right) - \frac{2GM}{r}\right]$$

$$\dot{r} = \frac{c^2}{\epsilon}\left(1 - \frac{2r_g}{r}\right)\sqrt{\left(\frac{\epsilon^2 - c^4}{c^2}\right) - \left[\frac{l^2}{r^2}\left(1 - \frac{2r_g}{r}\right) - \frac{2GM}{r}\right]}$$

$$\dot{\varphi} = \frac{c^2}{\epsilon}\left(1 - \frac{2r_g}{r}\right)\frac{l}{r^2}$$

$$\epsilon = c^2\sqrt{\frac{p^2 - 4pr_g + 4r_g^2(1 - e^2)}{p^2 - r_g p(e^2 + 3)}}$$

$$l^2 = c^2\left[\frac{r_g p^2}{p - r_g(e^2 + 3)}\right]$$

$$\ddot{x}_i = -\frac{GMx_i}{r^3}\left(1 - \frac{2r_g}{r}\right) + \frac{2r_g\dot{x}_i\dot{r}}{r(r - 2r_g)} + \frac{r_g x_i\dot{r}^2}{(r - 2r_g)r^2} - \frac{2r_g x_i\dot{\varphi}^2}{r}$$

$$r = \sqrt{x^2 + y^2 + z^2}, \; r\dot{r} = x\dot{x} + y\dot{y} + z\dot{z},$$

$$r^4\dot{\varphi}^2 = (x\dot{y} - y\dot{x})^2 + (x\dot{z} - z\dot{x})^2 + (z\dot{y} - y\dot{z})^2$$

Add the hydrodynamics :

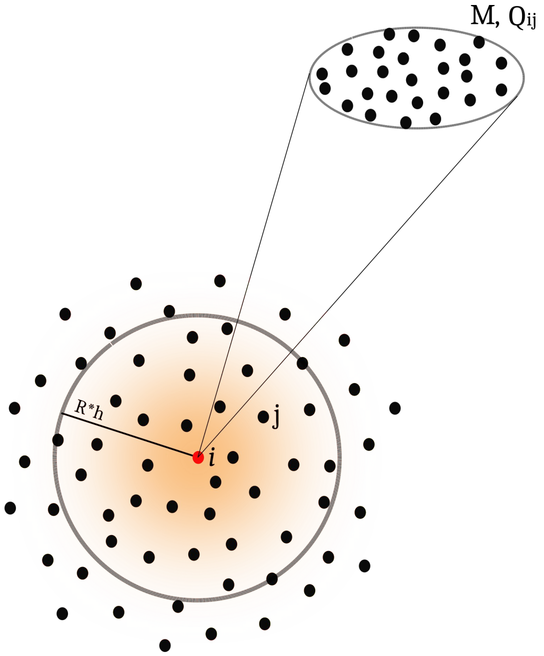
$$\frac{\partial\vec{v}}{\partial t} + \left(\vec{v}.\,\vec{\nabla}\right)\vec{v} + \frac{1}{\rho}\vec{\nabla}P + \vec{\nabla}\phi = 0$$

A few of words on SPH …

- An infinitesimal fluid element ≈ a smoothed particle
- Mesh-free Lagrangian method. No grids, no boundaries.
- Automatic conservation of energy, momentum (Ideally).

- Hydrodynamic properties and short range forces are evaluated from neighbours.
- Binary tree reduces interactions $\mathcal{O}(\mathcal{N}^2) \rightarrow \mathcal{O}(\mathcal{N}\log_2\mathcal{N})$
- Far field gravity is calculated using multipole moments of particle groups/ nodes.

- Temporal evolution is done using time reversal, symplectic leapfrog integrator.
- Individual time-stepping ensures that each particle evolves on its own time scale.
- Particle is “woken up” to react to its rapidly moving neighbours.

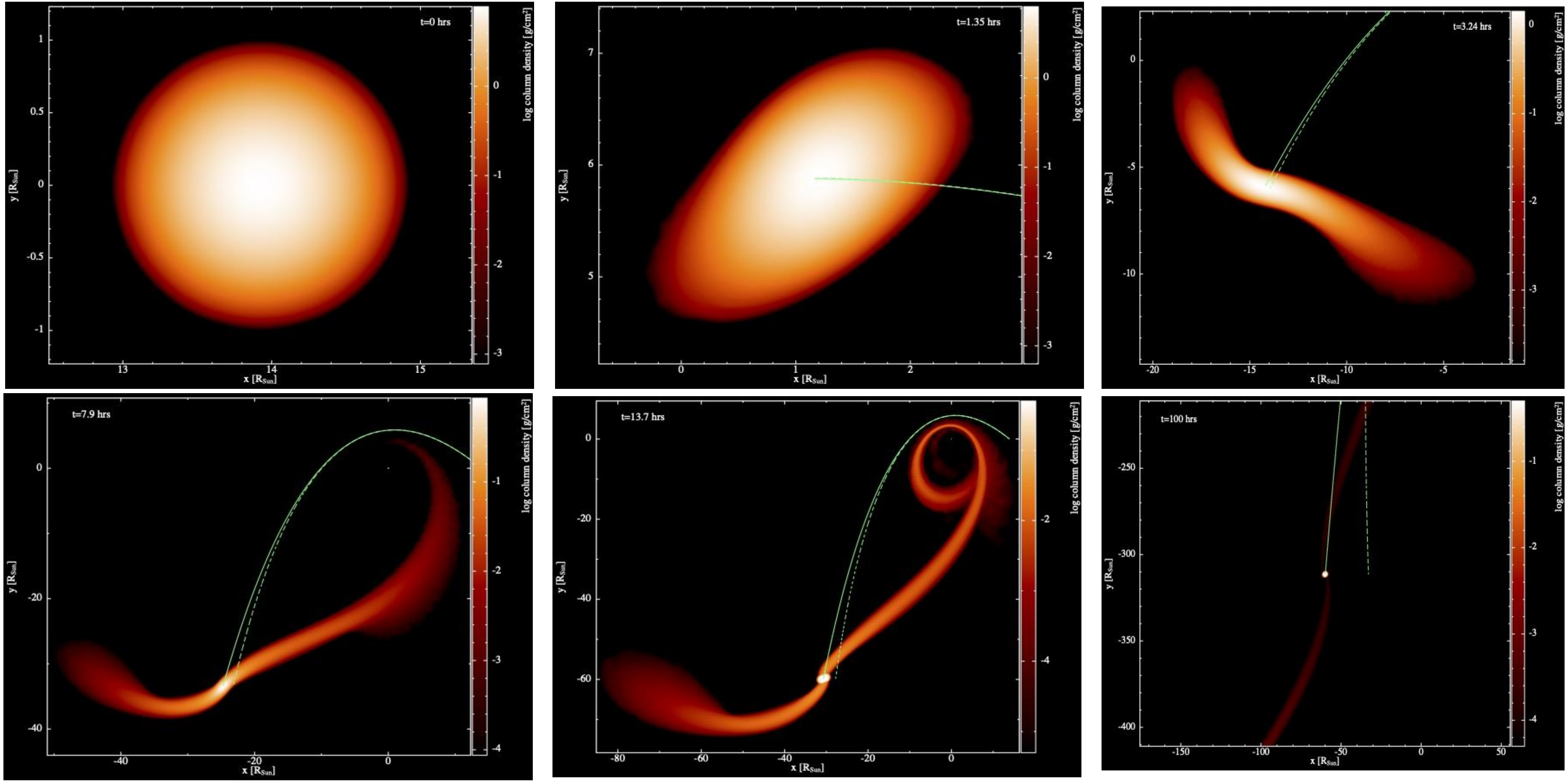


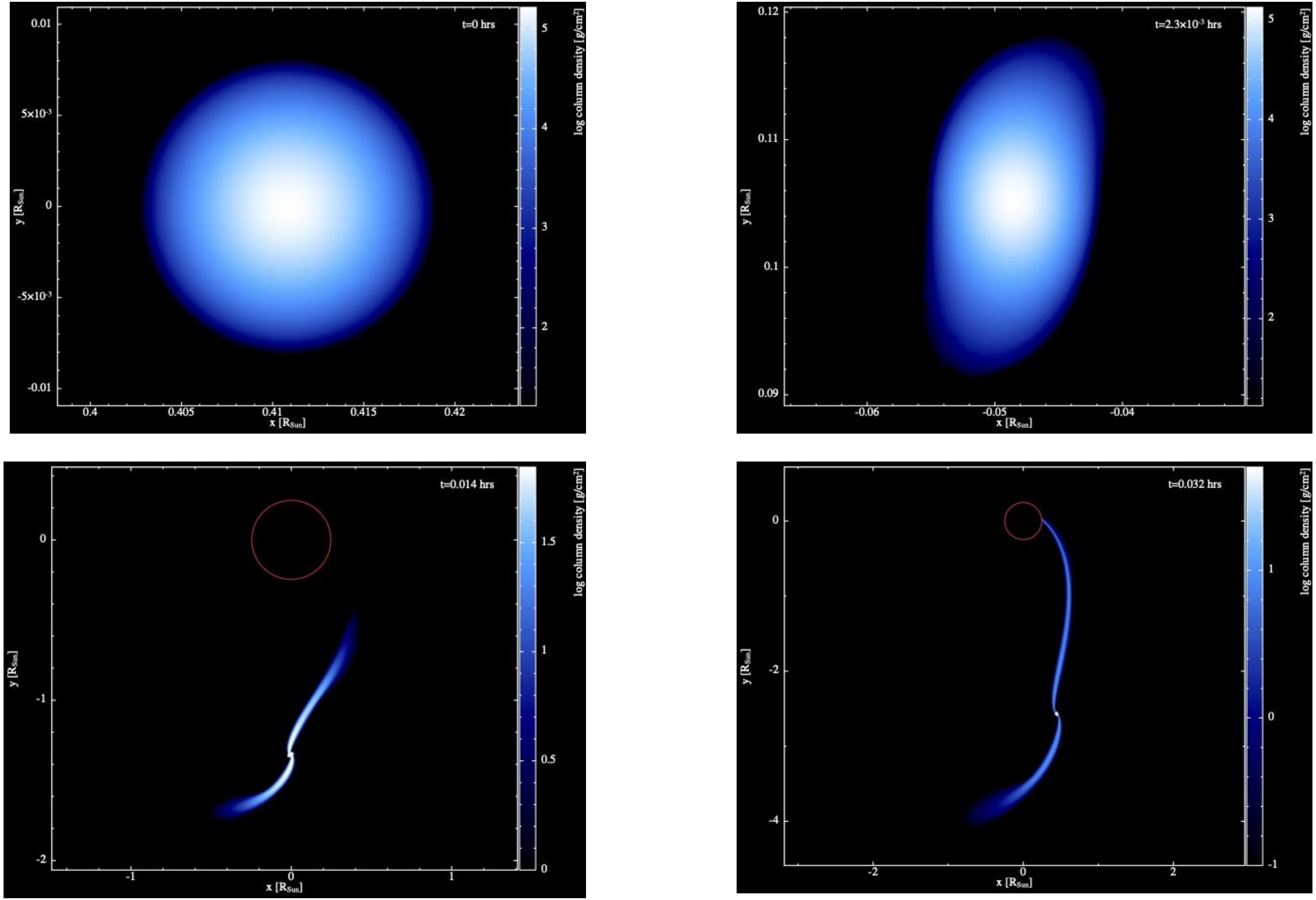
- We use a home grown numerical SPH code.
- It is written in C. For data analysis, we use Python, Mathematica, SPLASH
- Parallelized using OpenMP and MPI
- We work with 0.5 million particles, with convergence tests being done using 1 million particles
- Simulations are carried out at the PARAM Sanganak facility at IIT Kanpur as part of the National Supercomputing Mission, Govt. of India.

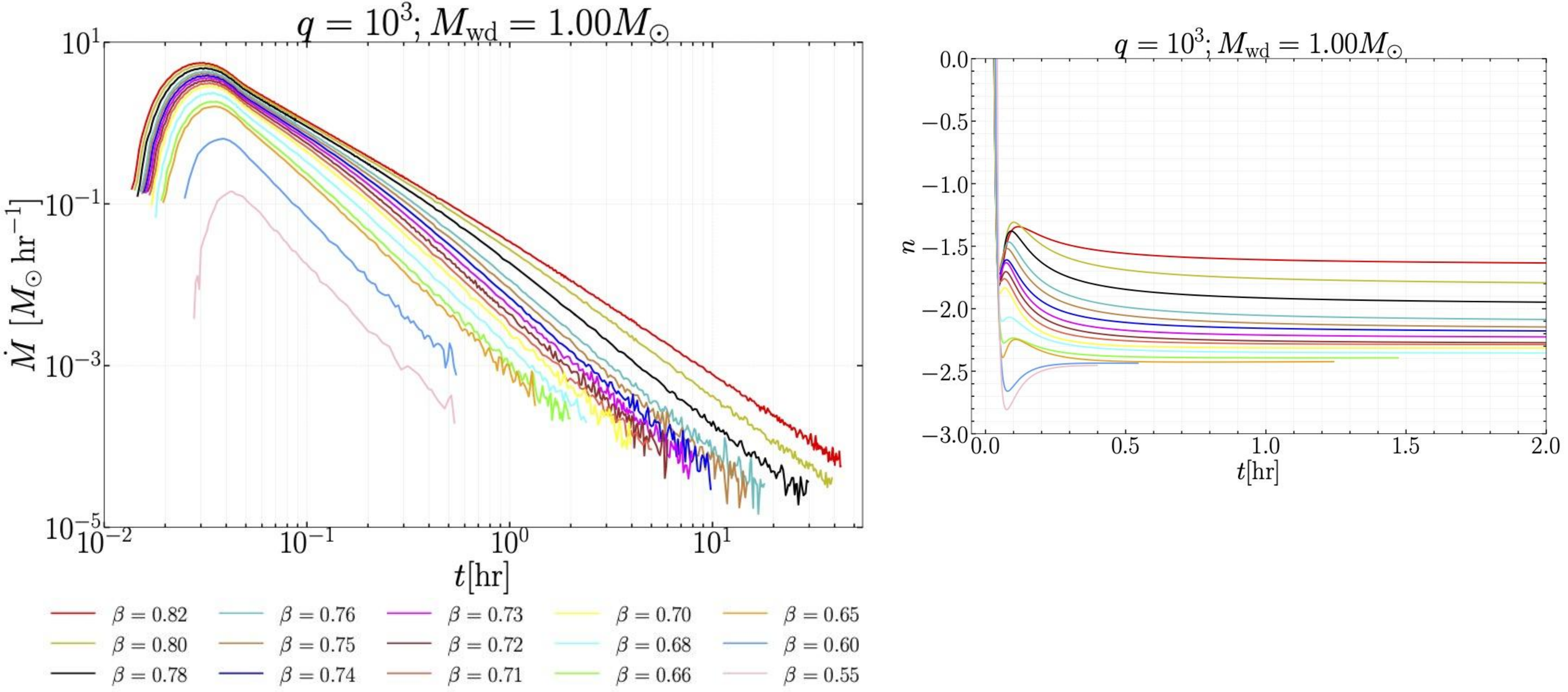
The first SPH code (1982) by Nolthenius and Katz used 40 particles to study $1\,M_\odot$ star disrupted by $10^4\,M_\odot$ BH.

Bicknell and Gingold (1983) used 500 particles to study $1\,M_\odot$ star disrupted by $10^5\,M_\odot$ BH.

Deviation in stellar trajectory due to asymmetric partial tidal disruption : first look : solid green : actual COM




t=0 hrs
log column density [g/cm²]
y [R_Sun]
x [R_Sun]
0.01
5×10⁻³
0
-5×10⁻³
-0.01
0.4
0.405
0.41
0.415
0.42
5
4
3
2
t=2.3×10⁻³ hrs
0.12
0.11
0.1
0.09
-0.06
-0.05
-0.04
t=0.014 hrs
-1
-2
1
1.5
0.5
t=0.032 hrs
-4
-1



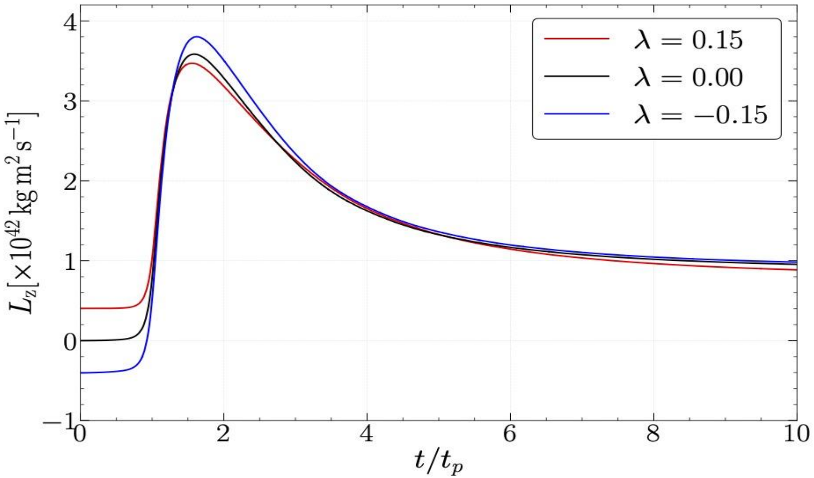
$q = 10^3; M_{\rm wd} = 1.00 M_\odot$
$\dot{M}\ [M_\odot\,{\rm hr}^{-1}]$
$t[{\rm hr}]$
$q = 10^3; M_{\rm wd} = 1.00 M_\odot$
n
$t[{\rm hr}]$
$\beta = 0.82$
$\beta = 0.80$
$\beta = 0.78$
$\beta = 0.76$
$\beta = 0.75$
$\beta = 0.74$
$\beta = 0.73$
$\beta = 0.72$
$\beta = 0.71$
$\beta = 0.70$
$\beta = 0.68$
$\beta = 0.66$
$\beta = 0.65$
$\beta = 0.60$
$\beta = 0.55$

Stars can also have large spins. Most WDs spin slowly with spin period of several hours. But –

- 2020: WD in the cataclysmic variable system spinning with 29.6 s.
- 2022: WD in the cataclysmic variable system spinning with 24.9 s.
- 2021 : Isolated WD spinning with 70 s.

Theoretically a WD can have a very small spin period, of the order of a few seconds.

Recent observations also indicate that IMBHs can be rapidly rotating with a > 0.97



Turns out that there is interesting physics when rotating stars meet Kerr black holes.

One reason is that what we call as the impact parameter changes with spin. Recall :

$$R_t = R_\star \times \left(\frac{M}{M_\star}\right)^{1/3}, \beta = \frac{R_t}{R_p}$$

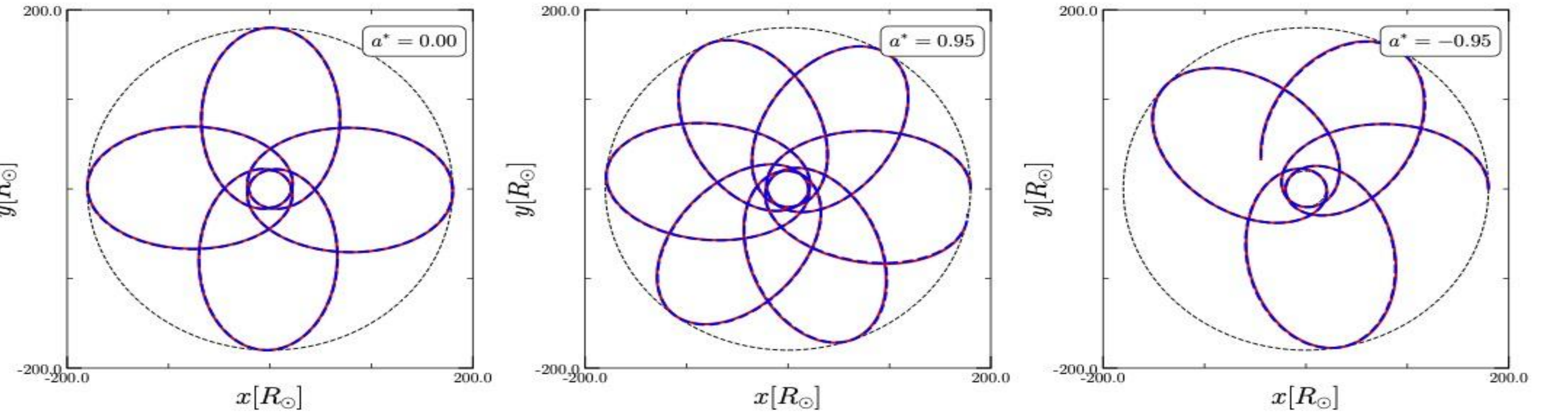
The situation if one includes stellar spin and Kerr black holes :

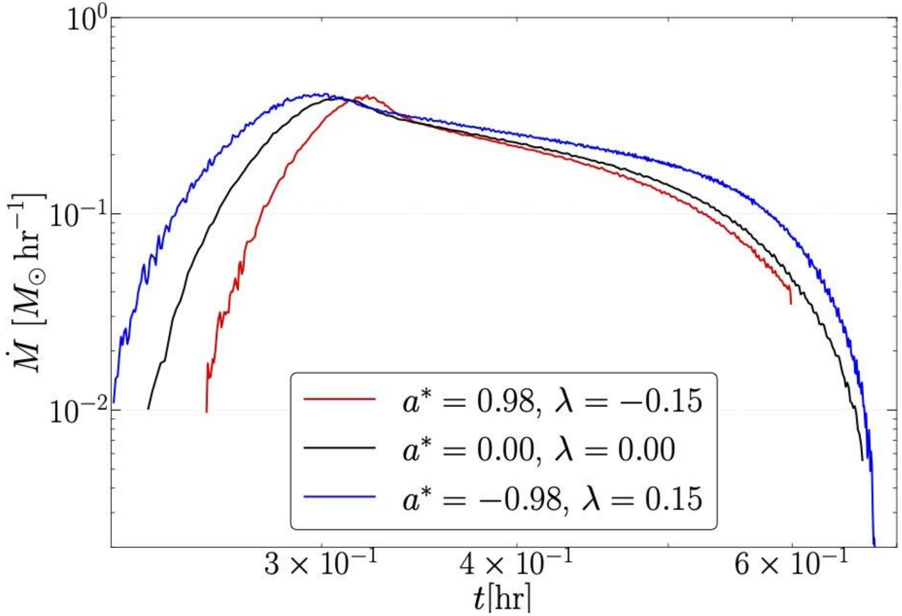
$$\frac{dU^\nu}{d\tau} = -\frac{1}{\rho\,\omega}\frac{\partial P}{\partial x^\mu}(U^\mu U^\nu + g^{\mu\nu}) - \Gamma^\nu_{\lambda\mu}U^\lambda U^\mu$$

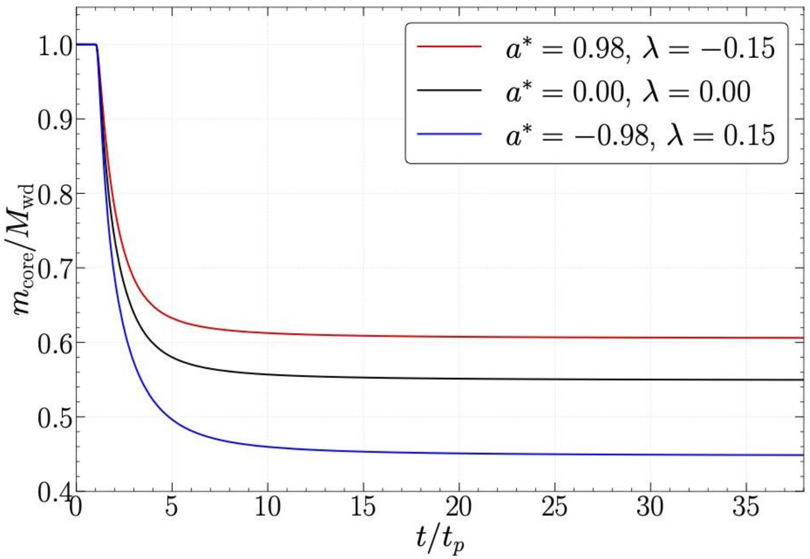
$$\frac{d^2x^i}{dt^2} = -(g^{i\lambda} - \dot{x}g^{0\lambda})\left[\frac{1}{\Gamma^2\rho\omega}\frac{\partial P}{\partial x^\lambda} + \left(\frac{\partial g_{\mu\lambda}}{\partial x^\sigma} - \frac{1}{2}\frac{\partial g_{\mu\sigma}}{\partial x^\lambda}\right)\dot{x^\mu}\dot{x^\sigma}\right]$$

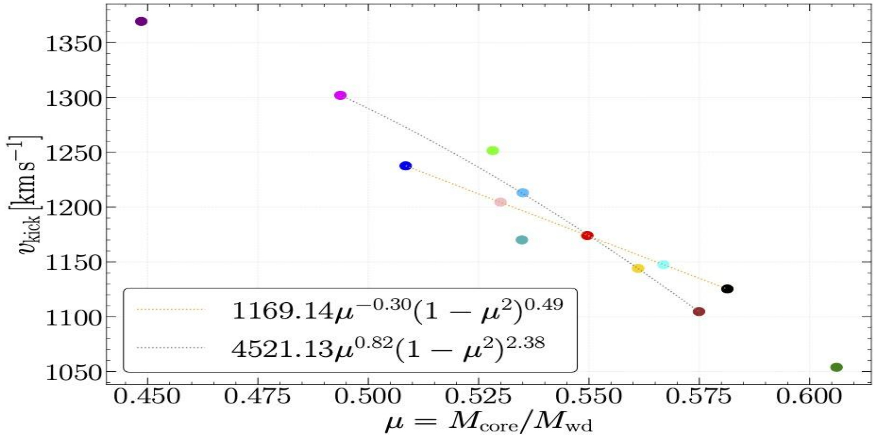
$$ds^2 = -\left(1 - \frac{2GMr}{c^2\,\Sigma}\right)dt^2 - \frac{4GMra\sin^2\theta}{c\,\Sigma}dtd\phi + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{2GMra^2\sin^2\theta}{c^2\,\Sigma}\right)\sin^2\theta d\phi^2$$

$$\Sigma = r^2 + a^2\cos^2\theta,\ \Delta = r^2 + a^2 - 2GMr/c^2,\ a = J/Mc$$









0.3 solar mass White Dwarf
Stellar spin period 5 minutes
10000 solar mass Kerr Black Hole
Minimum (periapsis) distance 39 Schwarzschild radii.

Significant amount of data over next 5-6 years

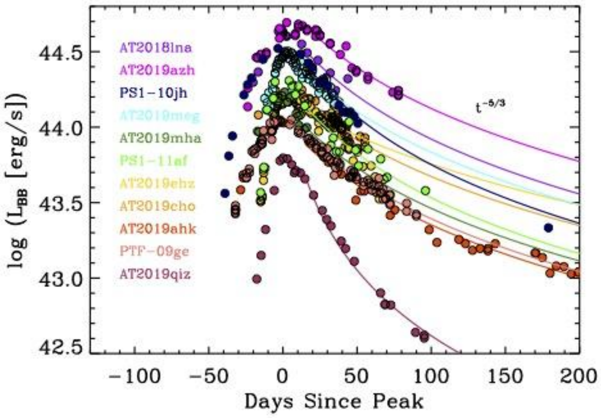


Image Taken from Gazeri, arXiv:2104.14580 [astro-ph.HE]

For the future :

What happens to phenomena like resonance locking for near extremal Kerr black holes ?

How do disruptions happen near wormhole throats ?

What happens when a star is disrupted very close to the event horizon ? Particularly important for White Dwarfs with Quantum Mechanics at play.